A graph theory of rook placements

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We define a new graph structure, called rook equivalence graphs. The vertices of a rook equivalence graph correspond to the Ferrers boards in a rook equivalence class and the edges are definide by transferring cells from one column of a board to another. We can classify whether a rook equivalence graph will be connected or not by examining the entries in the root vector of any board in the equivalence class. Using this classification, we demonstrate that complete bipartide graphs and cycle graphs cannot be rook equivalence graphs unless they are also complet graphs, K_2 and K_3 respectively. Furthermore, all complete graphs can be obtained as rook equivalence graphs. The construction and classification of connected rook equivalence graphs can also be accomplished for m-level rook placements, defined by Briggs and Remmel.

References

[1] K. Briggs and J. Remmel. m-rook numbers and a generalization of a formula of Frobenius to $C_m \wr \mathcal{S}_n$. *J. Combin. Theory Ser. A.*, 2006: 1138–1171.