Generalized Symmetric Edge Polytopes

Rob Davis Colgate University

Based on joint work with Akihiro Higashitani

The symmetric edge polytope (SEP) of a (finite, undirected) graph is a centrally symmetric lattice polytope whose vertices are defined by the edges of the graph. SEPs have been studied extensively in the past twenty years due to their connections to commutative algebra, their applications in the Kuramoto model for systems of interconnected oscillators, and more. Recently, D'Alí, Juhnke-Kubitzke, and Koch generalized the definition of an SEP to regular matroids, as these are the matroids that can be characterized by totally unimodular matrices. Generalized SEPs are known to have symmetric Ehrhart h^*-polynomials, and Ohsugi and Tsuchiya conjectured that (ordinary) SEPs have nonnegative γ-vectors.

In this talk, we give all necessary background and present additional extensions of properties known to hold for SEPs. We also show that generalized SEPs are not necessarily γ-nonnegative by providing explicit examples in all dimensions 10 and greater. However, these polytopes appear to be "nearly" γ-nonnegative in the sense that, by deleting exactly two elements from the matroid, one obtains SEPs for graphs that are γ-nonnegative. For those SEPs, we present a formula for their h^*-vectors and conjecture a formula for their γ-vectors.

We also discuss how this work is a consequence of a chance encounter between two postdocs at Michigan State University.

References

[1] Robert Davis and Akihiro Higashitani. On the Ehrhart Theory of Generalized Symmetric Edge Polytopes. *arXiv:2401.03383*