Inequalities for f^*-vectors of lattice polytopes

Danai Deligeorgaki KTH Royal Institute of Technology

Based on joint work with with Matthias Beck, Max Hlavacek and Jerónimo Valencia-Porras

The Ehrhart polynomial $\mathrm{ehr}_P(n)$ of a lattice polytope P counts the number of integer points in the n-th integral dilate of P. Ehrhart polynomials of polytopes are often described in terms of the vector of coefficients of $\mathrm{ehr}_P(n)$ with respect to different binomial bases, under which they have non-negative coefficients. Such vectors give rise to the h^* and f^*-vector of P, which coincide with the h and f vectors of a regular unimodular triangulation of P, whenever it exists. In particular, f^*-vectors were introduced in 2012 by Felix Breuer as the coefficients of $\mathrm{ehr}_P(n)$ expressed in the basis $\binom{n-1}{0}, \binom{n-1}{1}, \binom{n-1}{2}, \ldots$. We view examples of f^*-vectors of lattice polytopes, including a family of simplices whose f^*-vectors are not unimodal. Even though f^*-vectors of lattice polytopes are not necessarily unimodal, there are several interesting inequalities that can hold among their coefficients. These inequalities resemble striking similarities with existing inequalities for the coefficients of f-vectors of simplicial polytopes; e.g., the first half of the f^*-coefficients increases and the last quarter decreases. Even though f^*-vectors of polytopes are not always unimodal, there are several families of polytopes that carry the unimodality property. We also show that for any polytope with a given Ehrhart h^*-vector, there is a polytope with the same h^*-vector whose f^*-vector is unimodal.