Totally symmetric self-complementary plane partition matrices and related polytopes

Vincent Holmlund   North Dakota State University

Based on joint work with Jessica Striker

Plane partitions in the totally symmetric self-complementary symmetry class (TSSCPP) are known to be equinumerous with $n \times n$ alternating sign matrices, but no explicit bijection is known. In this paper, we give a bijection from these plane partitions to $\{0, 1, -1\}$-matrices we call magog matrices, some of which are alternating sign matrices. We explore enumerative properties of these matrices related to natural statistics such as inversion number and number of negative ones. We then investigate the polytope defined as their convex hull. We show that all the magog matrices are extreme and give a partial inequality description. Finally, we define another TSSCPP polytope as the convex hull of TSSCPP boolean triangles and determine its dimension, inequalities, vertices, and facets.