

Splines on Cayley Graphs of the Symmetric Group

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A spline is an assignment of polynomials to the vertices of a polynomial-edge-labeled graph, where the difference of two vertex polynomials along an edge must be divisible by the edge label. We consider spline modules where the underlying graph is the Cayley graph of a symmetric group constructed from a generating set of transpositions. These spline modules are generalizations of the Goresky-Kottwitz-MacPherson (GKM) construction for equivariant cohomology rings of the flag, Hessenberg, and permutohedral varieties. Each graded piece of these spline modules admits a graded S_n representation via the dot action. In the more restrictive GKM case, the character of this S_n representation is the chromatic symmetric function of a particular graph. In our more general setting, we present a combinatorial criteria for determining when graded pieces of the dot action representation are trivial, and a formula for the linear graded piece that applies for all generating sets.