Chow rings and augmented Chow rings of uniform matroids and their q-analogues

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Chow rings and augmented Chow rings of matroids play important roles in the celebrated proofs of two long-standing conjectures: (1) the Adiprasito-Huh-Katz proof of the Heron-Rota-Welsh Conjecture and (2) the Braden-Huh-Matherne-Proudfoot-Wang proof of the Dowling-Wilson Top-Heavy Conjecture. These two matroid invariants have since been extensively studied. In 2021, Hameister, Rao, and Simpson [2] gave a nice combinatorial interpretation of the Hilbert series of the Chow ring of the q-uniform matroid in terms of permutations and the q-Eulerian polynomials studied by Shareshian and Wachs. We present an analogous interpretation for the augmented Chow ring in terms of decorated permutations and q-binomial Eulerian polynomials. We also obtain the symmetric function analogs of the above results by studying the $\mathfrak{S}_n$-representations on Chow rings and augmented Chow rings of uniform matroids.

Our proof relies on a Feichtner-Yuzvinsky type basis for the augmented Chow ring of a matroid (introduced in our previous work [3] and in independent work of Eur, Huh and Larson [1]). This basis is also used to obtain closed form formulas for the Hilbert series of the augmented Chow ring of the uniform matroid evaluated at -1. These are analogous to our simplification of formulas of Hameister, Rao, and Simpson for the Chow ring.

References

[1] C. Eur, J. Huh, M. Larson. Stellahedral geometry of matroids. *Forum Math. Pi*, 11 2023: e24.

[2] T. Hameister, S. Rao, C. Simpson. Chow rings of vector space matroids. *J. Comb.*, 12(1) 2021: 55–83.

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