Penney's game for permutations

Yixin (Kathy) Lin Dartmouth College

Based on joint work with Sergi Elizalde

We consider the permutation analogue of Penney's game for words. Two players, in order, each choose a permutation of length $k \geq 3$; then a sequence of independent random values from a continuous distribution is generated, until the relative order of the last k numbers coincides with one of the chosen permutations, making that player the winner. We compute the winning probabilities for all pairs of permutations of length 3, showing that, as in the original version for words, the game is non-transitive. We give some formulas to compute the winning probabilities more generally, and we conjecture a winning strategy for the second player when k is arbitrary.

We also consider a Markov chain version of Penney's game for permutations. We generate permutations of length k using a transition matrix that goes from one pattern to another with probability $1/k$, where k is the length of two patterns, if the relative order of the last $k-1$ numbers of the first pattern is the same as the first $k-1$ numbers of the second pattern. We give a formula to compute the winning probabilities of all pairs of permutations of any length, and the expected time to see any permutations for the first time.