A rooted variant of Stanley's chromatic symmetric function

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Based on joint work with Greg Warrington

In 1995, Richard Stanley introduced the chromatic symmetric function of a graph, which is a multivariable generalization of the classical chromatic polynomial that encodes information about proper colorings of the graph. There exist non-isomorphic graphs with the same chromatic symmetric function, but Stanley conjectured that any two trees with the same chromatic symmetric function must be isomorphic. This conjecture, which is still open, stimulated an enormous amount of research in the area of graph colorings (including Gebhard and Sagan's noncommutative analogue of Stanley's chromatic functions). Here, we study variations of the chromatic symmetric function for rooted graphs, in which the root vertex is required to either use or avoid a specified color. We prove the rooted version of Stanley's conjecture: two rooted trees are isomorphic if their rooted chromatic polynomials are equal. Moreover, a certain one-variable specialization still contains enough information to distinguish rooted trees. We also prove the irreducibility of Stanley's chromatic function (specialized to finitely many variables) and some of its variations, using a novel combinatorial application of Eisenstein's Criterion.