

Finding Needles in a Haystack: Parking Functions, Fubini Rankings, and Boolean Intervals in the Weak Order of \mathfrak{S}_n

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Based on joint work with Jennifer Elder, Pamela E. Harris, and Jan Kretschmann

Let \mathfrak{S}_n denote the symmetric group and let $W(\mathfrak{S}_n)$ denote the weak order of \mathfrak{S}_n . Through a surprising connection to a subset of parking functions, which we call *unit Fubini rankings*, we provide a complete characterization and enumeration for the total number of Boolean intervals in $W(\mathfrak{S}_n)$ and the total number of Boolean intervals of rank k in $W(\mathfrak{S}_n)$. Furthermore, for any $\pi \in \mathfrak{S}_n$, we establish that the number of Boolean intervals in $W(\mathfrak{S}_n)$ with minimal element π is a product of Fibonacci numbers.

References

- [1] J. Elder, P. E. Harris, J. Kretschmann, and J. C. Martínez Mori. Boolean Intervals in the weak order of \mathfrak{S}_n . *arXiv:2306.14734*, 2023.