On ranked and bounded Kohnert posets

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Based on joint work with Laura Colmenarejo, Felix Hutchins, and Etienne Phillips

Kohnert polynomials form a large family of polynomials which are generalizations of both key and Schubert polynomials. Each Kohnert polynomial encodes a certain collection of diagrams which has a natural poset structure. It has been noted that such "Kohnert posets" are not usually "well-behaved". Here, we consider the question of identifying when Kohnert posets are well-behaved, focusing on the following two questions:

1. When are Kohnert posets bounded?

2. When are Kohnert posets ranked?

We present a sufficient condition guaranteeing when Kohnert posets are bounded and a necessary condition for when they are ranked. Moreover, we apply the aforementioned conditions to give a complete characterization of when the Kohnert posets associated with key diagrams are bounded and when they are ranked.

References

[1] A. Kohnert. Weintrauben, Polynome, Tableaux. *Bayreuth. Math. Schr.*, (38): 1–97, 1991.

[2] S. Armon, S. Assaf, G.H. Bowling, and H. Ehrhard. Kohnert's rule for flagged Schur modules. *J. Algebra*, 617(3): 352–381, 2023.

[3] S. Assaf. Demazure crystals for Kohnert polynomials. *Trans. Amer. Math. Soc.*, 375(3): 2147–2186, 2022.

[4] S. Assaf and D. Searles. Kohnert polynomials. *Exp. Math.*, 31(1): 93–119, 2022.

[5] L. Colmenarejo, F. Hutchins, N. Mayers, and E. Phillips On ranked and bounded Kohnert posets. arXiv preprint arXiv: 2309.007747, 2023.

[6] C. Kerr, N. Mayers, and N. Russoniello. Shellability of Kohnert posets. In preparation, +2024.