Pattern-avoiding polytopes and Cambrian lattices

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Based on joint work with Esther Banaian, Sunita Chepuri and Emily Gunawan

For each Coxeter element c in the symmetric group, we define a pattern-avoiding Birkhoff subpolytope whose vertices are the c-singletons. We show that the normalized volume of our polytope is equal to the number of longest chains in a corresponding type A Cambrian lattice. Our work extends a result of Davis and Sagan which states that the normalized volume of the convex hull of the 132 and 312 avoiding permutation matrices is the number of longest chains in the Tamari lattice, a special case of a type A Cambrian lattice. Furthermore, we prove that each of our polytopes is unimodularly equivalent to the order polytope of the heap of the c-sorting word of the longest permutation. This gives an affirmative answer to a generalization of a question posed by Davis and Sagan.