

Generalized parking functions and three-dimensional Lindström–Gessel–Viennot Lemma

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The “classical” parking functions of length n is counted by the formula $(n + 1)^{n-1}$. They corresponds bijectively to the standard Young tableaux (SYT) of skew-shapes $\alpha + 1^n/\alpha$, where α is any partition under $\lambda = (n - 1, \dots, 2, 1)$. There is a natural symmetric group action on these parking functions, where the orbit is counted by the Catalan number $\frac{1}{n+1} \binom{2n}{n}$. Then the Frobenius character of this action over all SYT of shapes $\alpha + 1^n/\alpha$ is given by the sum of all skew Schur functions $s_{(\alpha+1^n/\alpha)}(\mathbf{x})$. In this talk we generalize this notion to any partition λ and study the combinatorics of the generalized parking function by relating them to non-crossing lattice paths.