Difference d ascent sequences

Bruce E. Sagan Michigan State University

Based on joint work with Mark Dukes

Let $\alpha = a_1a_2\ldots a_n$ be a sequence of nonnegative integers. The ascent set of α, $\operatorname{Asc}\alpha$, consists of all indices k where $a_{k+1} > a_k$. An ascent sequence is α where the growth of the a_k is bounded by the elements of $\operatorname{Asc}\alpha$. These sequences were introduced by Bousquet-Mélou, Claesson, Dukes and Kitaev and have many wonderful properties. In particular, they are in bijection with unlabeled $(2+2)$-free posets, permutations avoiding a particular bivincular pattern, certain upper-triangular nonnegative integer matrices, and a class of matchings. A weak ascent of α is an index k with $a_{k+1} \geq a_k$ and weak ascent sequences are defined analogously to ascent sequences. These were studied by Bényi, Claesson and Dukes and shown to have similar equinumerous sets. Given a nonnegative integer d, we define a difference d ascent to be an index k such that $a_{k+1} > a_k - d$. We study the properties of the corresponding d-ascent sequences, showing that some of the maps from the weak case can be extended to bijections for general d while the extensions of others continue to be injective (but not surjective).