The Hilbert series of the superspace coinvariant ring

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Based on joint work with Brendon Rhoades

Let Ω_n denote "superspace" of rank n, i.e. the tensor product of the multivariate polynomial ring $\mathbb{C}[x_1, x_2, \ldots, x_n]$ with an exterior algebra of rank n. Ω_n can also be thought of as the ring of polynomial-valued holomorphic differential forms on complex n-space. Ω_n contains $\mathbb{C}[x_1, x_2, \ldots, x_n]$ and, much like $\mathbb{C}[x_1, x_2, \ldots, x_n]$, Ω_n carries an action of the symmetric group $\mathfrak{S}_n$. We let SI_n denote the ideal in Ω_n generated by the $\mathfrak{S}_n$-invariants in Ω_n with no constant term and study the quotient $SR_n := \Omega_n / SI_n$.

We give a formula for the double-graded Hilbert series of SR_n, which implies that the dimension of SR_n is the n^{th} ordered Bell number

$$\sum_{k=1}^{n} k! \operatorname{Stir}(n,k)$$

where $\operatorname{Stir}(n,k)$ is the Stirling number of the second kind. We also characterize the harmonic space attached to SR_n in terms of the Vandermonde determinant. Our results extend classical work on the coinvariants of the polynomial ring and prove conjectures of N. Bergeron, Li, Machacek, Sulzgruber, Swanson, Wallach, and Zabrocki. Finally, we show that the validity of a monomial basis for SR_n proposed by Sagan and Swanson is equivalent to the validity of a proposed basis for certain polynomial ring quotients.