Cyclic shuffle-compatibility: an algebraic approach

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Based on joint work with Jinting Liang and Bruce E. Sagan

A permutation statistic st is said to be shuffle-compatible if the distribution of st over the set of shuffles of two disjoint permutations π and σ depends only on $\mathrm{st}\,\pi$, $\mathrm{st}\,\sigma$, and the lengths of π and σ. Shuffle-compatibility is implicit in Stanley's early work on P-partitions [5], and was first explicitly studied by Gessel and Zhuang [3], who developed an algebraic framework for shuffle-compatibility centered around their notion of the shuffle algebra of a shuffle-compatible statistic. For a family of statistics called descent statistics, these shuffle algebras are isomorphic to quotients of the algebra of quasisymmetric functions.

Recently, Domagalski, Liang, Minnich, Sagan, Schmidt, and Sietsema [2] defined a version of shuffle-compatibility for statistics on cyclic permutations, and studied cyclic shuffle-compatibility through purely combinatorial means. In this talk, I will define the cyclic shuffle algebra of a cyclic shuffle-compatible statistic and present an algebraic framework for cyclic shuffle-compatibility in which the role of quasisymmetric functions is replaced by the cyclic quasisymmetric functions recently introduced by Adin, Gessel, Reiner, and Roichman [1]. This theory is used to provide explicit descriptions for the cyclic shuffle algebras of various cyclic permutation statistics, which in turn gives algebraic proofs for their cyclic shuffle-compatibility [4].

References

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